Macroeconomic Theory: Lecture 4

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Last week

- Models of consumption and saving
- Lab session on construction of the national accounts, looking at ESA 2010 (ESA 2013)
- Staring for a while at a transactions flow matrix of the Irish economy.

Today

- Infinite Horizon models
- Classical models of growth
- Eurostat and bulk download facility

The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.
- Assume utility from consumption is driven by Cobb Douglas-shaped functions of the form $U = C_0^{1-\beta} C_1^{\beta}$
- 0 means young, and 1 means old.
- The budget constraint in period 0 is $C_0 + K_1 \leq (1 + r_0 K_0)$.

Infinite Horizon Models

- Moving from 2 periods to 3 or 5 is trivial but a bit messy. A system like R or Mathematica can do it easily.
- Lots to be gained from examining the infinite horizon model, especially when invoking what is now called *Ricardian Equivalence*.
- Massive policy significance for these models, especially with respect to government debt dynamics, pensions policy, health care provision, and more.

Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$.
- The capitalist then chooses $\{C_t \geq 0, K_{t+1} \geq 0\}_{t=0}^{\infty}$ to maximise $(1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$ subject to $C_t + K_{t+1} \leq (1+r_r)K_t$ (assuming *Perfect Foresight*).
- Boiling this problem down using the same Lagrangian technique,
- you get that $1 + g_K = \frac{K_{t+1}}{K} = \beta(1+r)$.
- This is a very important equation and is usually called the *Cambridge Equation*.

Constant Saving Rate Model

- If we assume gross investment is a constant fraction of output, so $I = sX$
- Then remembering $\rho K = X$ and $r = v - \delta$, slot these into the Cambridge Equation to get essentially the same prediction as the neoclassical model: investment will be a constant proportion of output as long as the profit rate and productivity of capital don't change.

Savings and Growth Rates

- Exercise. Go to Eurostat. For an economy, Find Gross Investment, Divide it by Nominal GDP. What do you see?
- Exercise. In Eurostat file, Compute rates of saving I/X and Capital Accumulation I/K for the chosen economy.
- Fundamental issues in capitalist economies.
- The savings rate I/X gives us a sense of where an economy is on its growth/distribution schedule.
- A key equation to consider is $g_K + \delta = s\rho$.
- Exercise. Take 5 minutes and explain this equation to yourselves. Draw a picture of what it might look like.

Savings data

Figure 1: Personal Savings, US

Open questions

- We've relied on a two class assumption, capitalists and workers. What if there are capitalists, managers, and workers, or more? Discuss how this might change the models outlined above.

Story of a typical capitalist owning K

- Given the wage, choses the profit maximising technique of production
- Gets $v = \frac{x-w}{k} = \pi \cdot \rho$ as profit rate
- Allocate its wealth between consumption and capital: $C^c + K_{+1} = (1-\delta)K + v \cdot K$
- Assuming a constant share of wealth for consumption: $C^c = (1-\beta)(1+r)K$
- Assuming workers to consume all their wages: $g_K + \delta = \beta \cdot v - (1-\beta)(1-\delta)$

The classical Conventional Wage Model

- $w = x - v \cdot k$
- $c = x - (g_K + \delta)k$
- $\delta + g_K = \beta \cdot v - (1 - \beta)(1 - \delta)$
- $w = \overline{w}$
- Exercise: What happens when parameters change

Have a look at wage and productivity

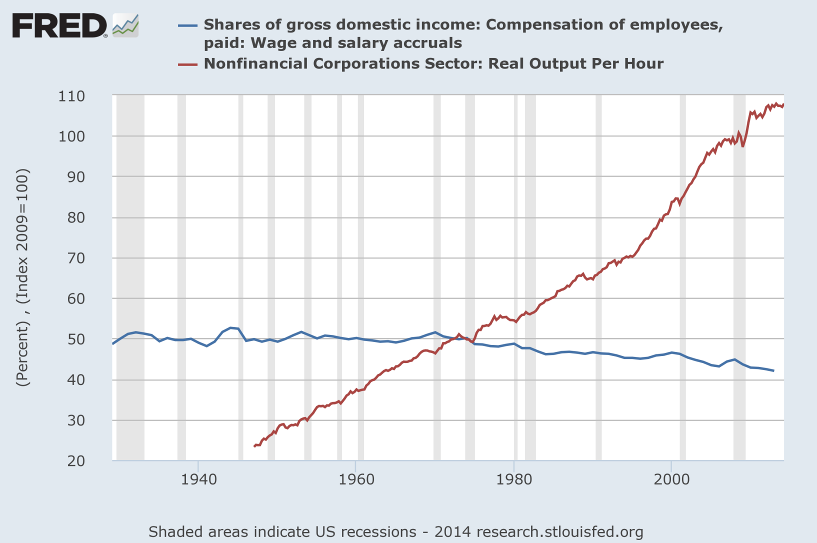

Figure 2: Wage share, output per hour, US

Have a look at wage and productivity

- If x keeps increasing, the wage share w/x tends towards 0
- The wage share on the other hand is fairly constant, need for a better model

The Classical conventional wage share model

- Assumes growth over time $x_t = x_0(1+\gamma)^t$
- Leads to $k_t = k_0(1+\gamma)^t$
- And to $w_t = w_0(1+\gamma)^t$
- Transformation into effective worker space leads to:
  - $\tilde{w} = \tilde{x} - v \cdot \tilde{k}$
  - $\tilde{c} = \tilde{x} - (g_K + \delta)\tilde{k}$
  - $\delta + g_K = \beta \cdot v - (1-\beta)(1-\delta)$
  - $\tilde{w} = (1-\overline{\pi})\tilde{x}$
- Further transformation into share space leads to:
  - $v = \pi \cdot \rho$
  - $g_k + \delta = s \cdot \rho$
  - $\delta + g_K = \beta \cdot v - (1-\beta)(1-\delta)$
  - $\pi = \overline{\pi}$

The classical models in the case of production functions

- In the case of Harrod-neural technical change, we have
  - $X_t = F_t(K, N) = F_0(K, (1 + \gamma)^t N)$
  - $\tilde{x} = F_t(\tilde{k}_t, 1) = F_0(\tilde{k}_t, 1)$
  - $\tilde{x} = f(\tilde{k}_t) \equiv F_0(\tilde{k}_t, 1)$
- Each technique of production can be represented by the income distribution schedule $\tilde{w} = \tilde{x} - v \cdot \tilde{k}$.
- For any effective wage $\tilde{w}$, there is only one profit maximising technique and we can thus determine it in most cases, given the wage share or effective wage.
- It is not the case for the Cobb-Douglas because the wage share is equal to $1 - \alpha$ for any wage

Classical model of growth with Full Employment

- Another closure than the conventional wage share hypothesis is assuming that the wage is the equilibrating factor between demand and supply of labor.
- Assumptions:
  - Supply of labor grows at rate n
  - Pure labor-saving technical change at rate γ
  - Effective labor supply grows at rate $n + \gamma$ (*natural rate of growth*)
- For full employment to be persistent, need : $N^s_{+1} = (1+n)N^s = (1+n)\frac{K}{k} = \frac{K_{+1}}{k_{+1}} = \frac{(1+g_K)K}{(1+\gamma)k}$
- Thus $1 + g_K = (1+n)(1+\gamma) \approx 1 + n + \gamma$ - Labor Market equation

Graphical representation

- Equations
  - $\tilde{w} = \tilde{x} - v \cdot \tilde{k}$
  - $\tilde{c} = \tilde{x} - (g_K + \delta)\tilde{k}$
  - $\delta + g_K = \beta \cdot v - (1 - \beta)(1 - \delta)$
  - $1 + g_K = (1 + n)(1 + \gamma)$
- See Mathematica file

Next Time

- Biased technical change, wage share changes, and the neoclassical growth model.
- Read GD, Chapters 7, 8.