# Macroeconomic Theory: Lecture 3

Antoine Godin Stephen Kinsella

# Last time:

- Looked at models of labour supply and production

# Last time:

- Looked at models of labour supply and production
- Saw a ‘lot’ of work on empirical macroeconomics.

# This time

- Models of consumption and saving, and classical models of growth

# This time

- Models of consumption and saving, and classical models of growth
- Lab session on construction of the national accounts, looking at ESA 2010 (ESA 2013)

# This time

- Models of consumption and saving, and classical models of growth
- Lab session on construction of the national accounts, looking at ESA 2010 (ESA 2013)
- Staring for a while at a transactions flow matrix of the Irish economy.

# Return to Piketty

- Recall the big P's two laws:

# Return to Piketty

- Recall the big P's two laws:
- An accounting identity (the share of capital in national income is the rate of return on capital times the ratio of capital to income, $\alpha = r\beta$, and

# Return to Piketty

- Recall the big P's two laws:
- An accounting identity (the share of capital in national income is the rate of return on capital times the ratio of capital to income, $\alpha = r\beta$, and
- a long-run equilibrium condition (the steady-state capital/income ratio is the savings rate divided by the economy-wide growth rate, $\beta = s/g$.

# Return to Piketty

- Recall the big P's two laws:
- An accounting identity (the share of capital in national income is the rate of return on capital times the ratio of capital to income, $\alpha = r\beta$, and
- a long-run equilibrium condition (the steady-state capital/income ratio is the savings rate divided by the economy-wide growth rate, $\beta = s/g$.
- The first can't help but be true, and the second tends to be of limited relevance if $s$ varies a lot, which it does.

# Data

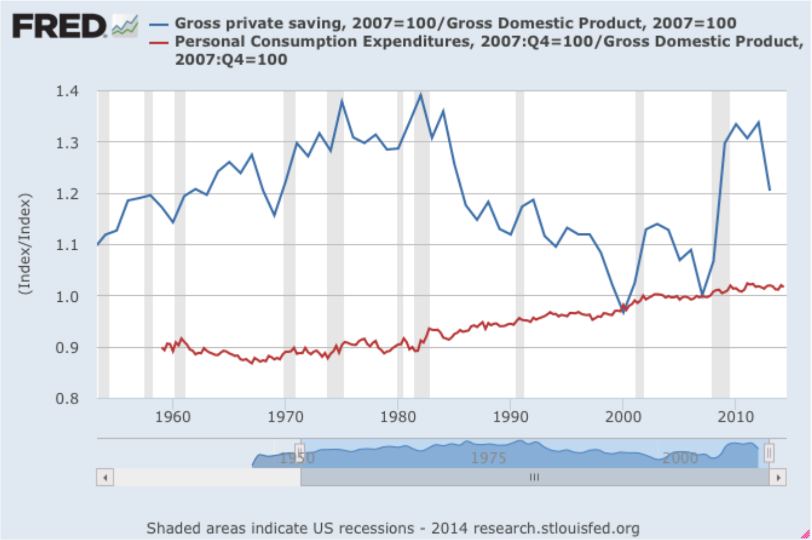


Figure 1: Consumption and Saving, US

# Workers and Saving

- Classicals assumed workers (as a class) saved basically nothing.

# Workers and Saving

- Classicals assumed workers (as a class) saved basically nothing.
- Life cycle theories of saving (Dusenberry, Modigliani) now dominate.

# Workers and Saving

- Classicals assumed workers (as a class) saved basically nothing.
- Life cycle theories of saving (Dusenberry, Modigliani) now dominate.
- In classical models, only the capitalist class does any saving.

# Workers and Saving

- Classicals assumed workers (as a class) saved basically nothing.
- Life cycle theories of saving (Dusenberry, Modigliani) now dominate.
- In classical models, only the capitalist class does any saving.
- Neoclassical models: consumption decisions are tradeoffs between consuming today and saving to consume in the future. The key problem is then to understand the intertemporal budget constraint.

# Workers and Saving

- Classicals assumed workers (as a class) saved basically nothing.
- Life cycle theories of saving (Dusenberry, Modigliani) now dominate.
- In classical models, only the capitalist class does any saving.
- Neoclassical models: consumption decisions are tradeoffs between consuming today and saving to consume in the future. The key problem is then to understand the intertemporal budget constraint.
- IBC gives rise to portfolio decisions, finance, and probabilistic modeling of uncertainty.

# The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.

# The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.
- Assume utility from consumption is driven by Cobb Douglas-shaped functions of the form $U = C_0^{1-\beta} C_1^{\beta}$

# The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.
- Assume utility from consumption is driven by Cobb Douglas-shaped functions of the form $U = C_0^{1-\beta} C_1^{\beta}$
- 0 means young, and 1 means old.

# The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.
- Assume utility from consumption is driven by Cobb Douglas-shaped functions of the form $U = C_0^{1-\beta} C_1^{\beta}$
- 0 means young, and 1 means old.
- The budget constraint in period 0 is $C_0 + K_1 \leq (1 + r_0 K_0)$.

# The 2 Period Model

- Super simple. Assume 2 periods, 0 and 1.
- Assume utility from consumption is driven by Cobb Douglas-shaped functions of the form $U = C_0^{1-\beta} C_1^{\beta}$
- 0 means young, and 1 means old.
- The budget constraint in period 0 is $C_0 + K_1 \leq (1 + r_0 K_0)$.
- Exercise. What is the budget constraint in period 1?

# Linear Programming (see handout and the growth theory notes)

- The households solve a linear program which we'll do on the board, subject to the intertemporal constraints.

# Linear Programming (see handout and the growth theory notes)

- The households solve a linear program which we'll do on the board, subject to the intertemporal constraints.
- The Lagrange multipliers of the system are $\partial L/\partial C_0 = \frac{1-\beta}{C_0} = \lambda_0$ and $\partial L/\partial C_1 = \frac{\beta}{C_1} = \lambda_1$

# Linear Programming (see handout and the growth theory notes)

- The households solve a linear program which we'll do on the board, subject to the intertemporal constraints.
- The Lagrange multipliers of the system are $\partial L / \partial C_0 = \frac{1-\beta}{C_0} = \lambda_0$ and $\partial L / \partial C_1 = \frac{\beta}{C_1} = \lambda_1$
- This means the marginal utility of consumption is always tending to increase.

# Linear Programming (see handout and the growth theory notes)

- The households solve a linear program which we'll do on the board, subject to the intertemporal constraints.
- The Lagrange multipliers of the system are $\partial L/\partial C_0 = \frac{1-\beta}{C_0} = \lambda_0$ and $\partial L/\partial C_1 = \frac{\beta}{C_1} = \lambda_1$
- This means the marginal utility of consumption is always tending to increase.
- The $K$ Lagrange multipliers are $\partial L/\partial K_1 = (1 + r_1)\lambda_1 = \lambda_0)$ and $\partial L/\partial K_2 = -\lambda_1 \leq 0$.

# Linear Programming (see handout and the growth theory notes)

- The households solve a linear program which we'll do on the board, subject to the intertemporal constraints.
- The Lagrange multipliers of the system are $\partial L/\partial C_0 = \frac{1-\beta}{C_0} = \lambda_0$ and $\partial L/\partial C_1 = \frac{\beta}{C_1} = \lambda_1$
- This means the marginal utility of consumption is always tending to increase.
- The $K$ Lagrange multipliers are $\partial L/\partial K_1 = (1 + r_1)\lambda_1 = \lambda_0)$ and $\partial L/\partial K_2 = -\lambda_1 \leq 0$.
- What variable does the capitalist choose, and why?

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.
- In the two period model $K_1 = \beta(1 + r_0 K_0)$ and

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.
- In the two period model $K_1 = \beta(1 + r_0 K_0)$ and
- For $C_0 = (1 - \beta)(1 + r_0)K_0$ with $K_0 = (1 + r_1 K_1) = \beta(1 + r_1)(1 + r_0)K_0$.

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.
- In the two period model $K_1 = \beta(1 + r_0 K_0)$ and
- For $C_0 = (1 - \beta)(1 + r_0)K_0$ with $K_0 = (1 + r_1 K_1) = \beta(1 + r_1)(1 + r_0)K_0$.
- Tells us the consumption in period 0 does not depends on the net profit rate in period 2.

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.
- In the two period model $K_1 = \beta(1 + r_0 K_0)$ and
- For $C_0 = (1 - \beta)(1 + r_0)K_0$ with $K_0 = (1 + r_1 K_1) = \beta(1 + r_1)(1 + r_0)K_0$.
- Tells us the consumption in period 0 does not depends on the net profit rate in period 2.
- Wealth and substitution effects of a change in net profit rates are equal and opposite in sign. Which is really cool.

# Cobb Douglass Demand system (see MMA demonstration)

- The solution in $\lambda$ space of these C and K equations is the Cobb Doublass demand system.
- In the two period model $K_1 = \beta(1 + r_0 K_0)$ and
- For $C_0 = (1 - \beta)(1 + r_0)K_0$ with $K_0 = (1 + r_1 K_1) = \beta(1 + r_1)(1 + r_0)K_0$.
- Tells us the consumption in period 0 does not depends on the net profit rate in period 2.
- Wealth and substitution effects of a change in net profit rates are equal and opposite in sign. Which is really cool.
- The capitalist's saving, $K_1$ is a constant fraction $\beta$ of her wealth ar the end of the period of some amount $(1 + r_0 K)0$.

# Infinite Horizon Models

- Moving from 2 periods to 3 or 5 is trivial but a bit messy. A system like R or Mathematica can do it easily.

# Infinite Horizon Models

- Moving from 2 periods to 3 or 5 is trivial but a bit messy. A system like R or Mathematica can do it easily.
- Lots to be gained from examining the infinite horizon model, especially when invoking what is now called *Ricardian Equivalence*.

# Infinite Horizon Models

- Moving from 2 periods to 3 or 5 is trivial but a bit messy. A system like R or Mathematica can do it easily.
- Lots to be gained from examining the infinite horizon model, especially when invoking what is now called *Ricardian Equivalence*.
- Massive policy significance for these models, especially with respect to government debt dynamics, pensions policy, health care provision, and more.

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$.

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$.
- The capitalist then chooses $\{C_t \geq 0, K_{t+1} \geq 0\}_{t=0}^{\infty}$ to maximise $(1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$ subject to $C_t + K_{t+1} \leq (1 + r_r)K_t$.

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1-\beta)\sum_{t=0}^{\infty}\beta^t LnC_t$.
- The capitalist then chooses $\{C_t \geq 0, K_{t+1} \geq 0\}_{t=0}^{\infty}$ to maximise $(1-\beta)\sum_{t=0}^{\infty}\beta^t LnC_t$ subject to $C_t + K_{t+1} \leq (1+r_r)K_t$.
- Boiling this problem down using the same Lagrangian technique,

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \ldots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \ldots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1 - \beta) \sum_{t=0}^{\infty} \beta^t LnC_t$.
- The capitalist then chooses $\{C_t \geq 0, K_{t+1} \geq 0\}_{t=0}^{\infty}$ to maximise $(1 - \beta) \sum_{t=0}^{\infty} \beta^t LnC_t$ subject to $C_t + K_{t+1} \leq (1 + r_r)K_t$.
- Boiling this problem down using the same Lagrangian technique,
- you get that $1 + g_K = \frac{K_{t+1}}{K} = \beta(1 + r)$.

# Working IH model out

- Instead of $t = 0, 1$ now we let $t = 0, 1, 2, \dots$.
- Capitalist now makes a sequence of decisions $\{C_0, C_1, \dots\}$ which we call $\{C_t\}_{t=0}^{\infty}$
- If the utility function is Cobb Douglas then $U(\{C_t\}_{t=0}^{\infty}) = (1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$.
- The capitalist then chooses $\{C_t \geq 0, K_{t+1} \geq 0\}_{t=0}^{\infty}$ to maximise $(1-\beta)\sum_{t=0}^{\infty} \beta^t LnC_t$ subject to $C_t + K_{t+1} \leq (1+r_r)K_t$.
- Boiling this problem down using the same Lagrangian technique,
- you get that $1 + g_K = \frac{K_{t+1}}{K} = \beta(1+r)$.
- This is a very important equation and is usually called the *Cambridge Equation*.

# Constant Saving Rate Model

- If we assume gross investment is a constant fraction of output, so $I = sX$

# Constant Saving Rate Model

- If we assume gross investment is a constant fraction of output, so $I = sX$
- Then remembering $\rho K = X$ and $r = v - \delta$, slot these into the Cambridge Equation to get essentially the same prediction as the neoclassical model: investment will be a constant proportion of output as long as the profit rate and productivity of capital don't change.

# Constant Saving Rate Model

- If we assume gross investment is a constant fraction of output, so $I = sX$
- Then remembering $\rho K = X$ and $r = v - \delta$, slot these into the Cambridge Equation to get essentially the same prediction as the neoclassical model: investment will be a constant proportion of output as long as the profit rate and productivity of capital don't change.
- Exercise. Go to FRED. For the US economy, Find Gross Investment, Divide it by Nominal GDP. What do you see?

# Constant Saving Rate Model

- If we assume gross investment is a constant fraction of output, so $I = sX$
- Then remembering $\rho K = X$ and $r = v - \delta$, slot these into the Cambridge Equation to get essentially the same prediction as the neoclassical model: investment will be a constant proportion of output as long as the profit rate and productivity of capital don't change.
- Exercise. Go to FRED. For the US economy, Find Gross Investment, Divide it by Nominal GDP. What do you see?
- Exercise. In FRED, Compute rates of saving $I/X$ and Capital Accumulation $I/K$ for the US.

# Savings and Growth Rates

- Fundamental issues in capitalist economies.

# Savings and Growth Rates

- Fundamental issues in capitalist economies.
- The savings rate $I/X$ gives us a sense of where an economy is on its growth/distribution schedule.

# Savings and Growth Rates

- Fundamental issues in capitalist economies.
- The savings rate $I/X$ gives us a sense of where an economy is on its growth/distribution schedule.
- A key equation to consider is $g_K + \delta = s\rho$.

# Savings and Growth Rates

- Fundamental issues in capitalist economies.
- The savings rate $I/X$ gives us a sense of where an economy is on its growth/distribution schedule.
- A key equation to consider is $g_K + \delta = s\rho$.
- Exercise. Take 5 minutes and explain this equation to yourselves. Draw a picture of what it might look like.

# Savings data


Figure 2: Personal Savings, US

# Open questions

- We've relied on a two class assumption, capitalists and workers. What if there are capitalists, managers, and workers, or more? Discuss how this might change the models outlined above.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p$.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p.$
- Accumulation of capital next period is given by $C^c + K_{+1} = (1-\delta)K + vK.$

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p$.
- Accumulation of capital next period is given by $C^c + K_{+1} = (1 - \delta)K + vK$.
- The 'new' Cambridge equation becomes $g_K + \delta = \beta\mu - (1 - \beta)(1 - \delta)$.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p$.
- Accumulation of capital next period is given by $C^c + K_{+1} = (1 - \delta)K + vK$.
- The 'new' Cambridge equation becomes $g_K + \delta = \beta\mu - (1 - \beta)(1 - \delta)$.
- Exercise. Describe this equation to an eight year old.

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p$.
- Accumulation of capital next period is given by $C^c + K_{+1} = (1 - \delta)K + vK$.
- The 'new' Cambridge equation becomes $g_K + \delta = \beta\mu - (1 - \beta)(1 - \delta)$.
- Exercise. Describe this equation to an eight year old.
- Handout: Let's look at the conventional wage model and what it means. (see table 6.1 of GD)

# Models of Economic Growth: Classical model

- The conventional wage model says that labour power is elastically supplied for a given warranted wage. That is, labour supply is horizontal given some $\bar{w}$.
- The capitalist gets a rental rate v from entrepreneurs.
- The residual of profit per unit of capital is equivalent to the profit share in output times the output capital ratio, -Or: $v = \frac{x-w}{k} = (1 - \frac{w}{x})\rho = \pi p$.
- Accumulation of capital next period is given by $C^c + K_{+1} = (1-\delta)K + vK$.
- The 'new' Cambridge equation becomes $g_K + \delta = \beta\mu - (1-\beta)(1-\delta)$.
- Exercise. Describe this equation to an eight year old.
- Handout: Let's look at the conventional wage model and what it means. (see table 6.1 of GD)
- Exercise. Go through the comparative dynamics of table 6.2, imagine the same situation playing out across the Irish economy today.

# The Wage Model with Technical Change

- Have to add steadily growing labour productivity via $x_t = x_0(1+\gamma)^t$.

# The Wage Model with Technical Change

- Have to add steadily growing labour productivity via $x_t = x_0(1+\gamma)^t$.
- This modifies the situation in table 6.1 markedly (see handout)

# The Wage Model with Technical Change

- Have to add steadily growing labour productivity via $x_t = x_0(1+\gamma)^t$.
- This modifies the situation in table 6.1 markedly (see handout)
- Now at full employment, the dynamics change again.

# The Wage Model with Technical Change

- Have to add steadily growing labour productivity via $x_t = x_0(1+\gamma)^t$.
- This modifies the situation in table 6.1 markedly (see handout)
- Now at full employment, the dynamics change again.
- Why look at full employment examples?

# The Wage Model with Technical Change

- Have to add steadily growing labour productivity via $x_t = x_0(1+\gamma)^t$.
- This modifies the situation in table 6.1 markedly (see handout)
- Now at full employment, the dynamics change again.
- Why look at full employment examples?
- Because then the capital growth rate $g_K = n + \gamma$, the Cambridge equation determines the profit rate and the growth/distribution schedule determines the wage/consumption schedule.

# Lab time: The construction of the national accounts, and a look at ESA 2010.

- To your computers!

# Next Time

- Biased technical change, wage share changes, and the neoclassical growth model.

# Next Time

- Biased technical change, wage share changes, and the neoclassical growth model.
- Read GD, Chapters 7, 8.