Stock Flow Consistent Models
An Introduction to Theory and Technique
Session 2

Eugenio Caverzasi
eu.caver@gmail.com

University of Pavia

Institute for New Economic Thinking - Young Scholars Initiative
Trento Festival of Economics

Outline

Where do the SFC models come from?

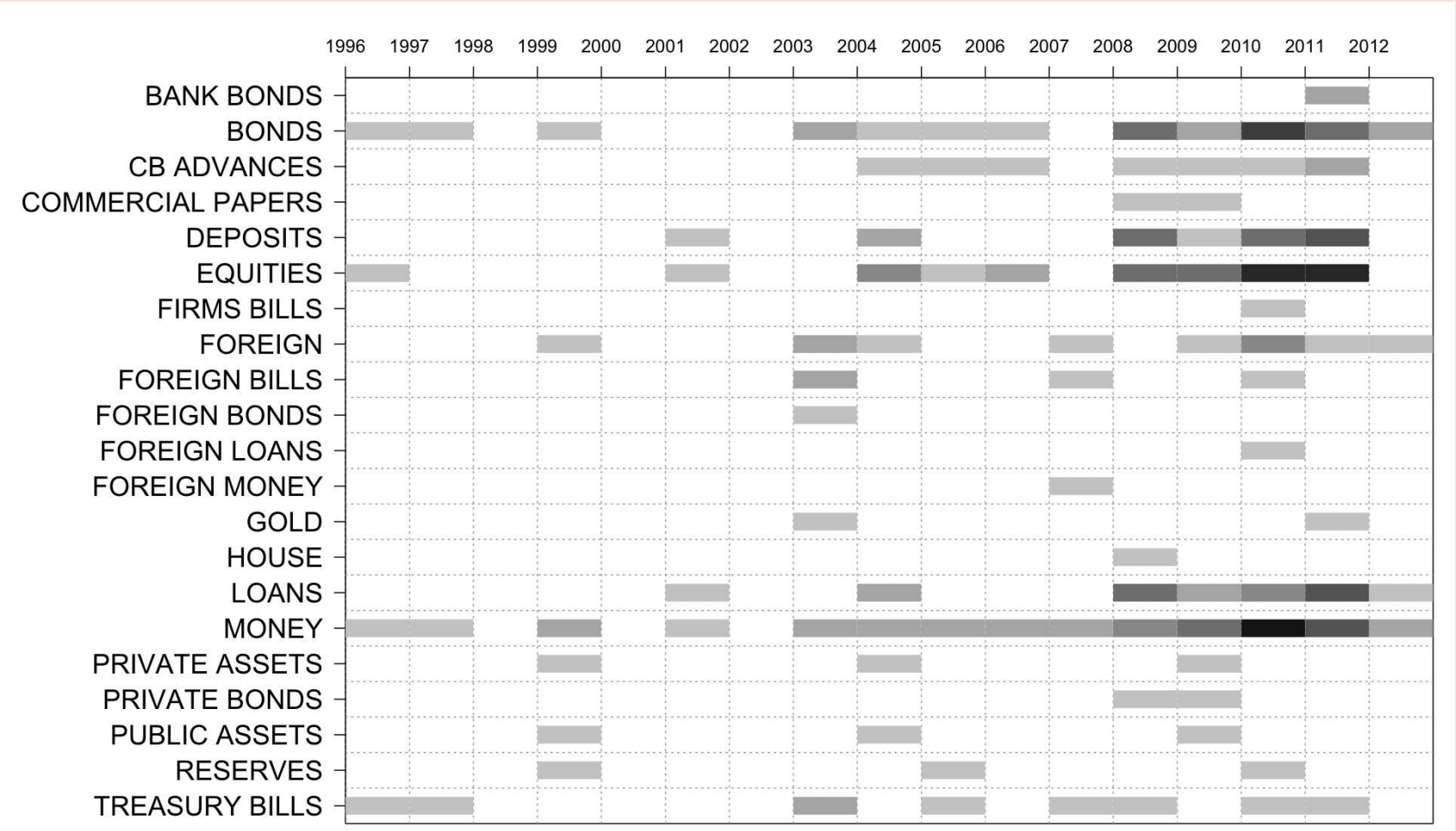
A very brief historical excursus

Three authors played a crucial role:

1. Morris A. Copeland (1949):
   - *Social Accounting for Moneyflows*
2. JamesTobin
   - Backus et al. (1980): arguably first complete SFC (matrix approach to accounting + closure).
   - Tobin (1982): Nobel lecture, in part a SFC Manifesto (1. Precision regarding time; 2. Tracking of stocks; 3. Several assets and rates of return; 4. Modeling of financial and monetary policy operations; 5. Walras's Law and adding up constraints.)
3. Wynne Godley
   - formalization and development, thanks to a more appropriate economic approach (money matters...)(among the other works see Godley and Lavoie, 2007):

Since then: great development $\rightarrow$

Assets through time (Caverzasi and Godin, 2012)



The darker the more papers used the asset

What are we talking about?

Which SFC are the components of an SFC model?

1. accounting part: matrix approach
2. behavioral equations

Accounting

Rules: consistency (wrt stock and flows, within and between)

Accounting

Rules: consistency (wrt stock and flows, within and between)

- someone's asset is someone else's liability AND someone inflow is someone else's outflow
  - *quadruple entry system* Copeland (1949)
- budget constraint for each individual sector and for the economy as a whole ("Walras' law and adding up constraint" Tobin 1982 or "budget constraint or system-wide consistency requirement" Godley and Lavoie 2007)

The matrix approach

1. Aggregate balance sheet: starting stocks of the economy.
2. Transaction flows: all the flows of the economy.
3. Flows of funds: how all flows end up in new stocks. End of the current period's stock = starting stocks of the following period.

NB 2 and 3 often presented jointly

Accounting part 1

THE INITIAL STOCKS: the aggregate balance sheet

Tab.1 Aggregate Balance Sheets. A (+) sign before a variable denotes an asset while a (-) sign denotes a liability

	Households	Firms	Banks	Gov.	Tot
Bank Deposits	+CA		-CA		0
Bank Loans		-L	+L		0
Capital		+K			+K
Net worth	Vh	Vf	Vr	Vg	V

Accounting part 2

CURRENT TRANSACTIONS: the transaction flows

Tab. 2 Current Transactions: (+) sign denotes receipt, (-) sign denotes a payment

	Households	Firms		Banks	Gov.	Tot
		current	capital			0
Consumption	-C	+C				0
Investment		$+\Delta K$	$-\Delta K$			0
Memo: Final Sales at market prices = pX = C + I = W + P						
Wages	+W	-W				0
Interests on L		$-rlL_{t-1}$		$+rlL_{t-1}$		0
Interests on CA	$+rcCA_{t-1}$			$-rcCA_{t-1}$		0
Dividends	+Ff	-Ff				0
Totals	SavH	Fu	$-\Delta K$	SavB	SavG	SAV

Accounting part 3

THE FLOW OF FUNDS: from the flows to the end of the period's stocks

Tab.3 Flow of Funds:

(+) sign denotes sources of funds, (-) denotes uses of funds

	Households	Firms	Banks	Gov.	Tot
Current Sav	+Sav H	$+Fu$	SavB	0	+SAV
Δ Bank Deposits	$-\Delta CA$		$+\Delta CA$		0
Δ Bank Loans		$+\Delta L$	$-\Delta L$		0
Δ Fixed K		$-\Delta K$			$-\Delta K$
Total	0	0	0	0	0
Net Worth	SAVH	Fu	Vb	0	SAV

MEMO: The net worth of a sector is increased by its current savings during the period, plus capital gains.

The equations: 2 steps

1. The accounting identities

All the identities and flows implied by the accounting
e.g. for firms (F= total profit)

$$F = +C + \Delta K + rc \cdot CA_{t-1} - W - rl \cdot L_{t-1} \quad (1)$$

$$Ff = F - Fu \quad (2)$$

2. The behavioral equations: economic theory comes into play

The closure: through theory we try to find an equation for each variable not directly determined by the accounting making theoretical assumptions on the behavior of the sectors.

$$Fu =$$

The equations: 2 steps

1. The accounting identities

All the identities and flows implied by the accounting
e.g. for firms (F= total profit)

$$F = +C + \Delta K + rc \cdot CA_{t-1} - W - rl \cdot L_{t-1} \quad (1)$$

$$Ff = F - Fu \quad (2)$$

2. The behavioral equations: economic theory comes into play

The closure: through theory we try to find an equation for each variable not directly determined by the accounting making theoretical assumptions on the behavior of the sectors.

$$Fu = \zeta F \quad (3)$$

$$C = W + ... \quad (4)$$

$$\Delta K = ... \quad (5)$$

How to use the components

We have: i) the accounting and ii) the equations (identities and behavioral).

According to

- Approach to the solution (simulated, analytical, theoretical)
- The data (real, fictitious, mixed)

We can have different approach to SFC models

1. *Theoretical models with a discursive solution*.
2. *Theoretical models solved via simulation*.
3. *Fully empirical*: the modelers not only estimate all their parameters, but also apply their model to predict variation in endogenous variables based on different scenarios, starting from the present state of economy.

+ several sub-groups: agent based vs aggregated, continuous vs discrete time... **?**[see][for some example]Caverzasi:2012b

A brief theoretical background

Money creation

1. *Outside money:* created when government pays for something, ceases to exist when government receives a payment from the public (e.g. taxes).
2. *Inside money:* created by private banks when they make loans, ceases to exist when debt is repaid.

Outline

Hypothesis 1

1. NO private money
   - no banks
   - no loans $\Rightarrow$ no interest payment
2. Closed
   - no import nor export
   - no capital flows
3. Pure labour economy
   - no K
   - no intermediate costs
4. No supply constraint of any kind
5. No inventories
6. Quantity adjustment mechanism $\Rightarrow$ S = D

Sectors

1. Households
   - buy consumption goods and pay taxes
   - get wages
   - accumulate assets
2. Producers
   - sell services or goods to households and govt
   - pay wages
3. Government
   - buy goods from firms
   - get taxes

Assets

- high powered money (cash)

The Model part 1: the accounting matrices

Starting point the aggregate balance sheet

Tab.1 Aggregate Balance Sheets

	Households	Firms	Government	Tot

The Model part 1: the accounting matrices

Starting point the aggregate balance sheet

Tab.1 Aggregate Balance Sheets

	Households	Firms	Government	Tot
Money Stock	$+H$		$-H$	0

(+) sign before a variable denotes an asset, (-) sign denotes a liability.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption				0
Govt. expenditure			$-G$	0
Wages				0
Taxes				0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption				0
Govt. expenditure		$+G$	$-G$	0
Wages				0
Taxes				0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption				0
Govt. expenditure		$+G$	$-G$	0
Wages		$-W \cdot N_d$		0
Taxes				0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption				0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes				0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

NB. *Quadruple-entry* system (Copeland, 1949): from one flow implies three more flows: govt. buys goods ($-G$), firms receive the payment ($+G$:) and use it to pay wages ($-WB$) to households ($+WB$).

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$			0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes	$-T$			0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$	$+C$		0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes	$-T$		$+T$	0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$	$+C$		0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes	$-T$		$+T$	0
Savings	S_h	0	S_g	0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$	$+C$		0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes	$-T_s$		$+T_d$	0
Savings	Sav_h		Sav_g	0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part 1: the accounting matrices

Tab.3 The Flow of Funds

	Households	Firms	Government	Tot
Current Savings	Sav_h		Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

The Model part 1: the accounting matrices

Tab.3 The Flow of Funds

	Households	Firms	Government	Tot
Current Savings	Sav_h		Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

Stock Flow Consistent models are *intrinsically dynamic* (see Macedo e Silva and Dos Santos, 2011)

The Model part 1: the accounting matrices

Tab.3 The Flow of Funds				
	Households	Firms	Government	Tot
Current Savings	Sav_h		Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

...the end of one period is the beginning of the following period.

Tab.1.b Aggregate Balance Sheets next period				
	Households	Firms	Government	Tot
Money Stock	$+H_{t+1}$		$-H_{t+1}$	0

(+) sign before a variable denotes an asset, (-) sign denotes a liability.

The Model part 1: the accounting matrices

Tab.3 The Flow of Funds				
	Households	Firms	Government	Tot
Current Savings	Sav_h		Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

Tab.1.b Aggregate Balance Sheets next period				
	Households	Firms	Government	Tot
Money Stock	$+H_{t+1}$		$-H_{t+1}$	0

(+) sign before a variable denotes an asset, (-) sign denotes a liability.

What about outside money?

Tab. 2 Current Transactions: (+) sign denotes receipt, (-) sign denotes a payment							
	Households	Firms		Banks		Gov.	Tot
		current	capital	current	capital		0
Cons	-C	+C					
Inv		$+\Delta K$	$-\Delta K$				0
G		+G				-G	0
Memo: Final Sales at market prices = pX = C + I +G = W + P							
Wages	+W	-W					0
Taxes	-T					+T	0
Int on L		$-rlL_{t-1}$		$+rlL_{t-1}$			0
Int on CA		$+rcCA_{t-1}$		$-rcCA_{t-1}$			0
Divs.	+Ff	-Ff					0
New L			$-\Delta L$		$+\Delta L$		0
Totals	SavH	Fu	$-\Delta K$	SavB		SavG	SAV

Firms receive a loan from banks and use it to pay wages...

Back to our model

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$	$+C$		0
Govt. expenditure		$+G$	$-G$	0
Wages	$+W \cdot N_s$	$-W \cdot N_d$		0
Taxes	$-T_s$		$+T_d$	0
Savings	Sav_h		Sav_g	0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

Tab.3 The Flow of Funds

	Households	Firms	Government	Tot
Current Savings	Sav_h		Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

The Model part 2: the behavioral equations

From assumption 4 (no supply constraints) and 6 (volume adjustment $S = D$)

$$C_s = C_d \tag{6}$$
$$G_s = G_d \tag{7}$$
$$T_s = T_d \tag{8}$$
$$N_s = N_d \tag{9}$$

The Model part 2: the behavioral equations

...

(10) Disposable income; (11) Taxes; (12) Consumption; (13) GDP; (14) employment

$$YD = W \cdot N_S - T \tag{10}$$

$$T = \theta \cdot W \cdot N_S \tag{11}$$

$$C = \alpha_1 \cdot YD + \alpha_2 \cdot H_{h-1} \tag{12}$$

$$Y = C_s + G_s \tag{13}$$

$$N = \frac{Y}{W} \tag{14}$$

The Model part 2: the behavioral equations

...

$$\Delta H_s = H_s - H_{s-1} = G - T \tag{15}$$

$$\Delta H_h = H_h - H_{h-1} = YD - C \tag{16}$$

watertight accounting $\Rightarrow$ Walrasian principle (n^{th} equation implied by the remaining n-1)

$$\Delta H_s = \Delta H_d \tag{17}$$

NB that is our *redundant equation:* when trying to compute a model, it is important to identify one and not include it in the computation, otherwise the model would be overdetermined. Remeber it can always be used to check if the model is correct (e.g. if $\Delta H_s \neq \Delta H_d$ we had a mistake)

Adding expectations: model SIMEX

Hyp: Consumption depends on *expected* NOT on actual income.

$\Rightarrow$: We discover the *buffer stock*

Supply equals demand = no news

$$C_s = C_d \tag{18}$$
$$G_s = G_d \tag{19}$$
$$T_s = T_d \tag{20}$$
$$N_s = N_d \tag{21}$$

Consumptions $\Leftarrow$ expectations

$$YD = W \cdot N_S - T \tag{22}$$
$$T = \theta \cdot W \cdot N_S \tag{23}$$
$$C = \alpha_1 \cdot YD^e + \alpha_2 \cdot H_{h-1} \tag{24}$$
$$Y = C_s + G_s \tag{25}$$
$$N = \frac{Y}{W} \tag{26}$$
$$YD^e = YD_{t-1} \tag{27}$$

Households can make a wrong estimate of their disposable income.
Hence the quantity of money held represents the adjusting mechanism (*i.e. buffer stock*)

...

$$\Delta H_s = H_s - H_{s-1} = G - T \tag{28}$$
$$\Delta H_h = H_h - H_{h-1} = YD - C_d \tag{29}$$
$$\Delta H_d = H_d - H_{h-1} = YD^e - C_d \tag{30}$$

Hence

$$H_h - H_d = YD - YD^e \tag{31}$$

SO: *if realized income is above expected income, households will hold the difference in the form of larger than expected cash money balances.*

The accounting. Q: what's the difference?

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption				0
Govt. expenditure		$+G$	$-G$	0
Wages				0
Taxes				0
Savings				0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

A: none

Tab.2 Current Transactions

	Households	Firms	Government	Tot
Consumption	$-C$	$+C$		0
Govt. expenditure		$+G$	$-G$	0
Wages	$+WN_s$	$-WN_d$		0
Taxes	$-T_s$		$+T_d$	0
Savings	Sav_h		Sav_g	0
Tot	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The Model part: the accounting matrices

Tab.3 The Flow of Funds

	Households	Firms	Government	Tot
Current Savings	Sav_h	0	Sav_g	0
Money Stock	$+\Delta H_h$		$-\Delta H_s$	0
Tot	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

$$\Delta H_h - \Delta H_d = YD - YD^e \tag{32}$$

$$\Delta H_h = H_h - H_{h-1} = YD - C_d \tag{33}$$

$$\Delta H_d = H_d - H_{h-1} = YD^e - C_d \tag{34}$$

Outline

Model PC

Novelties

- *government bills* (short term securities)
  - interest rate r
  - price $= 1$; fixed [$\Rightarrow$] NO capital gains
- *interest payments*
- *central bank*
  - distributes profits to government

The PC Model: the accounting matrices

Starting point the aggregate balance sheet

Tab.1 Aggregate Balance Sheets

	Households	Firms	Government	Central Banks	Tot
Money	$+H$		$-H$		0
Bills	B_h		$-B$	$+B_{cb}$	0
Balance (net worth)	$-V$		$+V$		0
$\sum$	0		0		0

(+) sign before a variable denotes an asset, (-) sign denotes a liability.

The PC Model: the accounting matrices

Tab.2 Current Transactions

	Households	Firms	Government	Central Bank	Tot
Cons.	$-C$	$+C$			0
Govt. expend.		$+G$	$-G$		0
Wages	$+W$	$-W$			0
Interests	$+r_{-1} \cdot B_{h-1}$		$-r_{-1} \cdot B_{-1}$	$+r_{-1} \cdot B_{cb-1}$	0
CB profits			$+F_{cb}$	$-F_{cb}$	0
Taxes	$-T$		$+T$		0
Savings	$Savh$		$Savg$		0
Tot	0	0	0	0	0

(+) sign denotes an inflow, (-) sign denotes an outflow.

The PC Model: the flow of funds

Starting point the aggregate balance sheet

Tab.1 Aggregate Balance Sheets

	Households	Firms	Government	Central Banks	Tot
Savings	$Savh$	0	$Savg$	0	0
Money	$-\Delta H$		$+\Delta H$		0
Bills	$-\Delta B_h$		$+\Delta B$	$-\Delta B_{cb}$	0
$\sum$	0	0	0	0	0

(+) sign denotes a use of funds, (-) sign denotes a source of funds.

The PC Model: the behavioral equations

We still assume for *perfect foresight*⇒ quantitative adjustment: S=D

GDP, disposable income and taxes

$$Y = C + G \tag{35}$$
$$YD = W - T + r \cdot B_{h-1} \tag{36}$$
$$T = \theta \cdot (Y + r \cdot B_{h-1}) \tag{37}$$

What causes a change in wealth is savings

Wealth and consumption

$$V = V_{-1} + (YD - C) \tag{38}$$
$$C = \alpha_1 \cdot YD + \alpha_2 \cdot V_{-1} \tag{39}$$

The Portfolio decision

How do households allocate their wealth?

They make a 2 steps decision

1. how much to save out of YD?
2. how to allocate their savings?

The Portfolio decision

How do households allocate their wealth?
They make a 2 steps decision

1. how much to save out of YD?
2. how to allocate their savings?

possible answers

1. *Quantity theory of money:* money balance linked to the flow of income.
2. More recently (linked to Keynesian liquidity preferences): money balance is some proportion of total wealth.
3. *Tobinesque portfolio approach* (Tobin 1969, Brainard and Tobin 1968): enclose transaction demand and liquidity preferences

...the Tobinesque portfolio approach...

$$\frac{H_h}{V} = (1 - \lambda_0) - \lambda_1 r + \lambda_2 \frac{YD}{V} \tag{40}$$

$$\frac{B_h}{V} = \lambda_0 + \lambda_1 r - \lambda_2 \frac{YD}{V} \tag{41}$$

some rules

- one proportion desired for each asset λ_0 and $(1 - \lambda_0)$
  - adding up constraint $\Rightarrow$ opposite signs of the parameters (hyp: consistent portfolio plans).
- the proportion are modulated by the rate of return of different assets (r, usually r^e since forward looking) and the disposable income to wealth (transaction demand)
  - if expectation mistakes $\Rightarrow$ buffer stock

remaining equations

$$\Delta H_h = \Delta B_h \tag{42}$$
$$\Delta B_s = B_s - B_{s-1} = (G + r_{-1} \cdot B_{s-1}) - (T + r_{-1} \cdot B_{cb-1}) \tag{43}$$
$$\Delta H_s = H_s - H_{s-1} = \Delta B_{cb} \tag{44}$$
$$\Delta B_{cb} = B_s - B_h \tag{45}$$
$$\tag{46}$$

Central Bank: residual buyer

redundant equation ($H_h = H_s$)

References I

David Backus, William C. Brainard, Gary Smith, and James Tobin. A Model of U.S. Financial and Nonfinancial Economic Behavior. *Journal of Money, Credit and Banking*, 12(2):259–293, May 1980.

E. Caverzasi and A. Godin. Stock-flow consistent modeling through the ages. *The Levy Economics Institute of Bard College Working Paper*, (745), January 2012.

Morris A. Copeland. Social Accounting for Moneyflows. *The Accounting Review*, 24(3):pp. 254–264, 1949. ISSN 00014826.

W. Godley and M. Lavoie. *Monetary Economics An Integrated Approach to Credit, Money, Income, Production and Wealth*. Palgrave MacMillan, New York, 2007.

Antonio Carlos Macedo e Silva and Cláudio H. Dos Santos. Peering over the Edge of the Short Period? The Keynesian Roots of Stock-Flow Consistent Macroeconomic Models. *Cambridge Journal of Economics*, 35(1):105–124, 2011.

References II

James Tobin. Money and Finance in the Macroeconomic Process. *Journal of Money, Credit and Banking*, 14(2):pp. 171–204, 1982. ISSN 00222879.