Stock Flow Consistent Models
An Introduction to Theory and Technique
Session 3

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Institute for New Economic Thinking -Young Scholars Initiative
Trento Festival of Economics

Outline

Outline

1. PK-SFC Package
2. Model SIM
   - 3.4 A numerical example and the standard Keynesian multiplier
   - 3.7 Expectations
   - Appendix 3.4 Government deficit and growth
3. Model PC
   - 4.4 Buffer stocks and expectations
   - 4.7 A government target for the debt to income ratio

Motivation

Allowing to simulate SFC models in an open source environment

- Very preliminary
- Only one numerical solver: Gauss-Seidel algorithm [Kinsella and O'Shea, 2010]
- Searching for Beta Tester (godin.antoine@gmail.com)
- http://www.antoinegodin.eu/sfc

Technical aspect

- R package
- EViews translator
- Read equation files
- Build model from console

Outline

Model SIM I

Transaction Flow Matrix

	Households	Production	Government	$\sum$
Consumption	$-C$	$+C$		0
Govt. expenditures		$+G$	$-G$	0
[Output]		$[Y]$		
Wages	$+WB$	$-WB$		0
Taxes	$-T$		$+T$	0
Savings	S_h	0	S_g	0
Change in money stock	$-\Delta H$		$+\Delta H$	0
$\sum$	0	0	0	0

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[Output]		$[Y]$		
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Change in money stock	$-\Delta H$		$+\Delta H$	0
$\sum$	0	0	0	0

Balance Sheet

	Households	Production	Government	Σ
Money	$+H$		$-H$	0

Model SIM II

sim.sfc

```
yd = w*n - t
t = theta*w*n
c = alpha1*yd + alpha2*h(-1)
h = h(-1) + yd - c
h_c = h_c(-1)+ g - t
y = c + g
n = y/w
```

Model SIM II

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h_c = h_c(-1)+ g - t
y = c + g
n = y/w
```

Loading the package and the model

```
>library(PKSFC)
>sim<-sfc.model("sim.sfc",modelName="sim")
```

Checking the model I

```
>sim<-sfc.check(sim,fill=T)
Initial value for h [0]?
1:
```

Checking the model I

```
>sim<-sfc.check(sim,fill=T)
Initial value for h [0]?
1:
Initial value for h_c [0]?
1:
One or more exogenous variables are not defined in the
following equations w * n - t
theta * w * n
alpha1 * yd + alpha2 * h_1
h_c_1 + g - t
c + g
y/w
do you want to insert these manually [Yes]/No?
1:
```

Checking the model II

```
Insert name, value and description (return after each
value)
1:
```

Checking the model II

```
Insert name, value and description (return after each
value)
1:
 Insert name, value and description (return after each
value)
1:  theta
2:  0.2
3:  tax rate
Are there other variables to add [Yes]/No?
1:
```

Parameter values

w	1	Wage rate	alpha1	0.6	Prop. to consume (income)
g	20	Govt expenditure	alpha2	0.4	Prop. to consume (wealth)

Checking the model III

```
Years are not set, do you want to inser these manually
[Yes]/No?
1:
```

Checking the model III

```
Years are not set, do you want to inser these manually
[Yes]/No?
1:
Insert initial period and final period (return after each
value)
1:  1945
2:  2010
```

Checking the model III

```
Years are not set, do you want to inser these manually
[Yes]/No?
1:
Insert initial period and final period (return after each
value)
1:  1945
2:  2010
>datasim<-simulate(sim)
```

Results

```
>years=c("1945","1946","1947","2010")
>variables=c("g","y","t","yd","c","h","h_c")
>round(t(datasim$baseline[years, variables]), digits=1)
```

	1945	1946	1947	2010
g	20	20.0	20.0	20
y	NA	38.5	47.9	100
t	NA	7.7	9.6	20
yd	NA	30.8	38.3	80
c	NA	18.5	27.9	80
h	0	12.3	22.7	80
h_c	0	12.3	22.7	80

Outline

Keynesian multiplier

Equations

$$C_d = \alpha_1 \cdot YD + \alpha_2 \cdot H_{-1} = \alpha_1 \cdot YD \tag{3.13}$$

$$Y = C + G = \alpha_1 \cdot Y \cdot (1 - \theta) + G$$

$$Y^{\star} = \frac{G}{1 - \alpha_1 \cdot (1 - \theta)} \tag{3.14}$$

Keynesian multiplier

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Short run vs. Long run

- (3.14) is the short run multiplier, depends on start-of-period stock values (i.e $H_{-1} = 0$)

Keynesian multiplier

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- Need to obtain the steady state to compute long-run multiplier

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Short run vs. Long run

- (3.14) is the short run multiplier, depends on start-of-period stock values (i.e $H_{-1} = 0$)
- Need to obtain the steady state to compute long-run multiplier
- Steady state: $Y^\star = \frac{G}{\theta} = \frac{20}{0.2} = 100$

Results

```
>years=c("1945","1946","1947","2010")
>variables=c("g","y","t","yd","c","h","h_c")
>round(t(datasim$baseline[years, variables]), digits=1)
```

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g	20	20.0	20.0	20
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Outline

Introducing expectation

Expected income

$$C_d = \alpha_1 \cdot YD^e + \alpha_2 \cdot H_{h,-1} \tag{3.7E}$$

$$\Delta H_d = YD^e - C_d \tag{3.18}$$

$$YD^e = YD_{-1} \tag{3.20}$$

Introducing expectation

Expected income

$$C_d = \alpha_1 \cdot YD^e + \alpha_2 \cdot H_{h,-1} \quad (3.7E)$$

$$\Delta H_d = YD^e - C_d \quad (3.18)$$

$$YD^e = YD_{-1} \quad (3.20)$$

R code

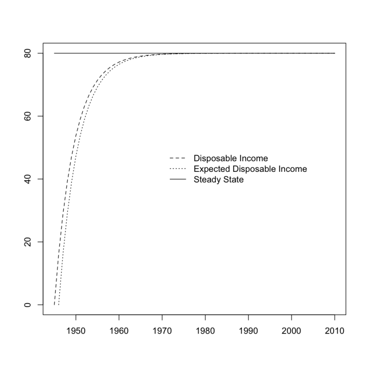
```
>simex_a<-sfc.addEqus(sim,list(
list(var="yde",eq="yd(-1)",desc="Expected income"),
list(var="h_d",eq="h(-1)+yde-c",desc="Expected wealth")))
>simex_a<-sfc.editEqu(simex_a,var="c",eq="alpha1*yde+alpha2*h(
>simex_a<-sfc.check(simex_a,fill=T)
```

Playing with expectations

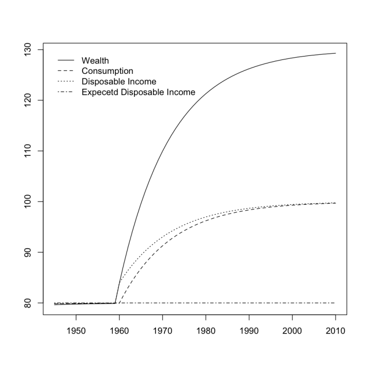
```
>timeline=seq(from=1945,to=2010)
>plot(timeline,datasimex$baseline[as.character(timeline),"yd"],type="l",
xlab="",ylab="",lty=2)
>lines(timeline,datasimex_a$baseline[as.character(timeline),"yde"],lty=3)
>lines(timeline,vector(length=length(timeline))+datasimex_a$baseline["2010","yd"])
>legend(x=1970,y=50,legend=c("Disposable Income","Expected Disposable
Income","Steady State"),lty=c(2,3,1),bty="n")
```

Results

$YD^e = YD(-1)$



$YD^e = YD^\star$



Outline

Growth I

Sources of growth in SIM

- Government expenditure: $G = G_{-1}(1 + gr_g)$
- Consumption: α_1 or α_2

Growth I

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- Consumption: α_1 or α_2

Growth steady state equations

$$\frac{\Delta H^\star}{Y^\star} = \frac{gr \cdot (1-\theta) \cdot (1-\alpha_1)}{gr + \alpha_2} \qquad \text{(A3.3.2)}$$

$$\frac{H^\star}{Y^\star} = \frac{(1+gr) \cdot (1-\theta) \cdot (1-\alpha_1)}{gr + \alpha_2} \qquad \text{(A3.3.3)}$$

$$\frac{V^T}{Y^\star} = \alpha_3 = \frac{1-\alpha_1}{\alpha_2} \qquad \text{(A3.3.4)}$$

$$\frac{H^\star}{YD^\star} = \frac{(1+gr) \cdot (1-\alpha_1)}{gr + \alpha_2} \qquad \text{(A3.3.5)}$$

Growth II

Source code

```
>simgr<-sfc.addEqu(sim,"g","g(-1)*(1+grg)","government
expenditures")
```

Growth II

Source code

```
>simgr<-sfc.addEqu(sim,"g","g(-1)*(1+grg)","government
expenditures")
```

Steady state values ($gr_g = 0\%$)

	$\frac{\Delta H^\star}{Y^\star}$	$\frac{H^\star}{Y^\star}$	$\frac{V^T}{Y^\star}$	$\frac{H^\star}{YD^\star}$
Value	0	0.8	1	1
Simulation	$5.2e^{-7}$	0.7999992	1	0.999999

Growth II

Source code

```
>simgr<-sfc.addEqu(sim,"g","g(-1)*(1+grg)","government
expenditures")
```

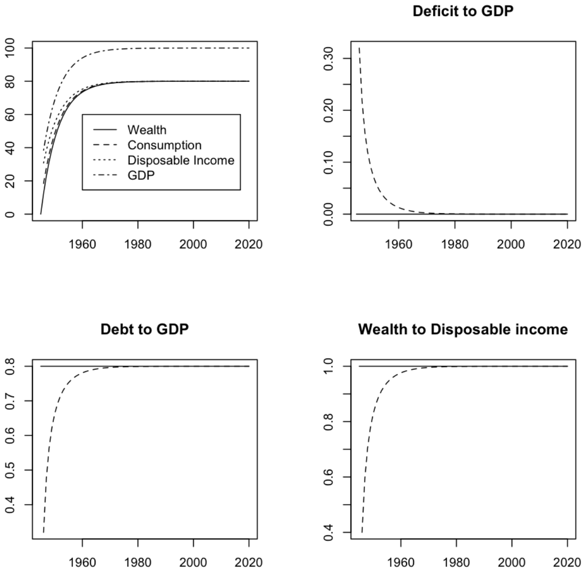
Steady state values ($gr_g = 0\%$)

	$\frac{\Delta H^\star}{Y^\star}$	$\frac{H^\star}{Y^\star}$	$\frac{V^T}{Y^\star}$	$\frac{H^\star}{YD^\star}$
Value	0	0.8	1	1
Simulation	$5.2e^{-7}$	0.7999992	1	0.999999

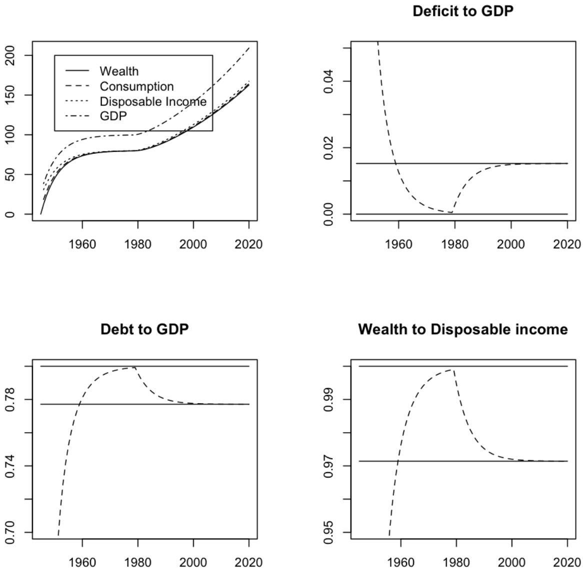
Steady state values ($gr_g = 2\%$)

	$\frac{\Delta H^\star}{Y^\star}$	$\frac{H^\star}{Y^\star}$	$\frac{V^T}{Y^\star}$	$\frac{H^\star}{YD^\star}$
Value	0.0152381	0.7771429	1	0.9714286
Simulation	0.01523065	0.777154	1	0.9714425

Results - No growth



Results - Growing expenditures



Outline

Model with Portfolio choice

Balance Sheet

	Households	Production	Government	Central Bank	Σ
Money	$+H$			$-H$	0
Bills	$+B_h$		$-B$	$+B_{cb}$	0
Balance	$-V$		$+V$		0
Σ	0		0	0	0

Model with Portfolio choice

Balance Sheet

	Households	Production	Government	Central Bank	Σ
Money	$+H$			$-H$	0
Bills	$+B_h$		$-B$	$+B_{cb}$	0
Balance	$-V$		$+V$		0
Σ	0		0	0	0

Portfolio equation

$$\frac{H_h}{V} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{4.6A}$$

$$\frac{B_h}{V} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD}{V}\right) \tag{4.7}$$

$$H_h = V - B_h \tag{4.6}$$

From model SIM to model PC

```
>pc<-sfc.addEqus(sim,list(
list(var="v",eq="v(-1)+yd-c",desc="households wealth"),
list(var="bs",eq="bs(-1)+g+r(-1)*bs(-1)-t-r(-1)*bcb(-1)",
desc="Supply of bonds"),
list(var="bcb",eq="bs-bh",desc="Bonds held by the central
bank"),
list(var="r",eq="r(-1)",desc="Interest rate"),
list(var="bh",eq="v*(lambda0+lambda1*r-lambda2*yd/v)",
desc="Bonds held by housholds")))
```

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```
>pc<-sfc.addEqus(sim,list(
list(var="v",eq="v(-1)+yd-c",desc="households wealth"),
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bank"),
list(var="r",eq="r(-1)",desc="Interest rate"),
list(var="bh",eq="v*(lambda0+lambda1*r-lambda2*yd/v)",
desc="Bonds held by housholds")))
>pc<-sfc.editEqus(pc,list(
list(var="yd",eq="y-t+r(-1)*bh(-1)"),
list(var="t",eq="theta*(y+r(-1)*bh(-1))"),
list(var="c",eq="alpha1*yd+alpha2*v(-1)"),
list(var="h",eq="v-bh",desc="cash holding"),
list(var="h_c",eq="h(-1)+bcb-bcb(-1)",desc="hidden
equation")))
```

From model SIM to model PC

```
>pc<-sfc.addEqus(sim,list(
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list(var="bs",eq="bs(-1)+g+r(-1)*bs(-1)-t-r(-1)*bcb(-1)",
desc="Supply of bonds"),
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bank"),
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list(var="bh",eq="v*(lambda0+lambda1*r-lambda2*yd/v)",
desc="Bonds held by housholds")))
>pc<-sfc.editEqus(pc,list(
list(var="yd",eq="y-t+r(-1)*bh(-1)"),
list(var="t",eq="theta*(y+r(-1)*bh(-1))"),
list(var="c",eq="alpha1*yd+alpha2*v(-1)"),
list(var="h",eq="v-bh",desc="cash holding"),
list(var="h_c",eq="h(-1)+bcb-bcb(-1)",desc="hidden
equation")))
>pc<-sfc.rmEqu(pc,7)# Employment equation
```

Outline

Adding expectations

Equation

$$C = \alpha_1 \cdot YD^e + \alpha_2 \cdot V_{-1} \tag{4.5E}$$

$$\frac{H_h}{V^e} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \tag{4.6E}$$

$$\frac{B_h}{V^e} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \tag{4.7E}$$

Adding expectations

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$$\frac{B_h}{V^e} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \tag{4.7E}$$

Buffer Stocks

"money balance are the element of flexibility in a monetary system of production"[Godley and Lavoie, 2007, p. 108]

Adding expectations

Equation

$$C = \alpha_1 \cdot YD^e + \alpha_2 \cdot V_{-1} \tag{4.5E}$$

$$\frac{H_h}{V^e} = (1 - \lambda_0) - \lambda_1 \cdot r + \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \tag{4.6E}$$

$$\frac{B_h}{V^e} = \lambda_0 + \lambda_1 \cdot r - \lambda_2 \cdot \left(\frac{YD^e}{V^e}\right) \tag{4.7E}$$

Buffer Stocks

"money balance are the element of flexibility in a monetary system of production"[Godley and Lavoie, 2007, p. 108]

$$H_d = V_{-1} + (YD^e - C) - B_h \tag{4.13}$$

$$H_h = V_{-1} + (YD - C) - B_h$$

Outline

Debt to GDP

Maastricht treaty

The reference values referred to [...] are:

- 3% for the ratio of the planned or actual government deficit to gross domestic product at market
- 60% for the ratio of government debt to gross domestic product at market prices.

Debt to GDP

Maastricht treaty

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- 60% for the ratio of government debt to gross domestic product at market prices.

Steady state

$$\frac{V^\star}{Y^\star} = \frac{\frac{1-\alpha_1}{\alpha_2}}{1 + \left[\frac{\theta}{1-\theta}\right] - r \cdot \left[(\lambda_0 + \lambda_1 \cdot r) \cdot \frac{1-\alpha_1}{\alpha_2} - \lambda_2\right]} \qquad (4.33)$$

Implementing the Maastricht treaty I

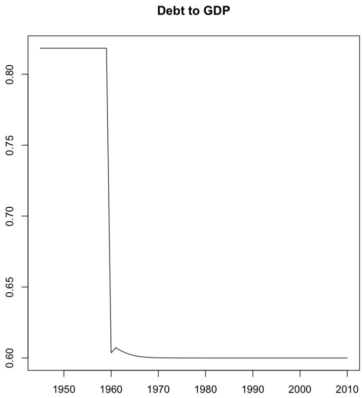
Assuming rational government

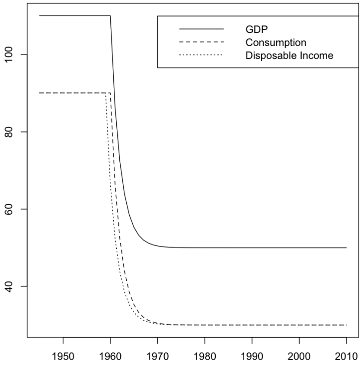
$$\theta = \frac{\frac{1-\alpha_1}{\alpha_2} \cdot \frac{1}{M} + r \cdot \left[(\lambda_0 + \lambda_1 \cdot r) \cdot \frac{1-\alpha_1}{\alpha_2} - \lambda_2 \right] - 1}{\frac{1-\alpha_1}{\alpha_2} \cdot \frac{1}{M} + r \cdot \left[(\lambda_0 + \lambda_1 \cdot r) \cdot \frac{1-\alpha_1}{\alpha_2} - \lambda_2 \right]}$$

Implementing the Maastricht treaty I

Assuming rational government

$$\theta = \frac{\frac{1-\alpha_1}{\alpha_2} \cdot \frac{1}{M} + r \cdot \left[(\lambda_0 + \lambda_1 \cdot r) \cdot \frac{1-\alpha_1}{\alpha_2} - \lambda_2 \right] - 1}{\frac{1-\alpha_1}{\alpha_2} \cdot \frac{1}{M} + r \cdot \left[(\lambda_0 + \lambda_1 \cdot r) \cdot \frac{1-\alpha_1}{\alpha_2} - \lambda_2 \right]}$$





Implementing the Maastricht treaty II

Assuming (somewhat) rational government

$$G = G_{-1}(1 + gr_g)$$

Implementing the Maastricht treaty II

Assuming (somewhat) rational government

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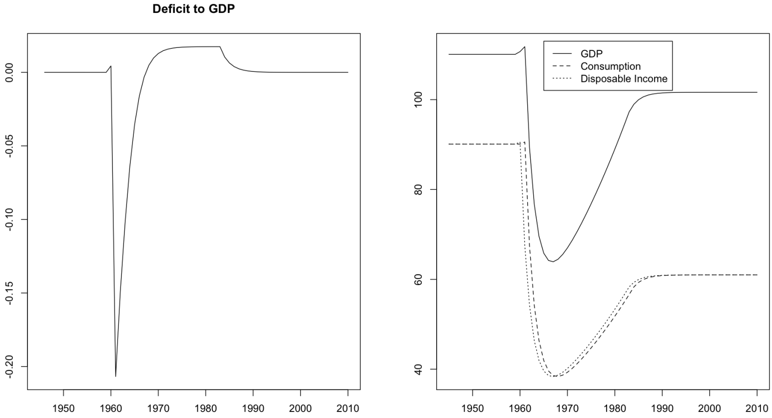


Implementing the Maastricht treaty III

Assuming (more) rational government

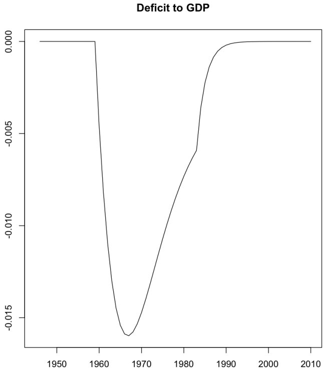
$$G = G_{-1}(1 + gr_g) \qquad gr_g = 3\%$$

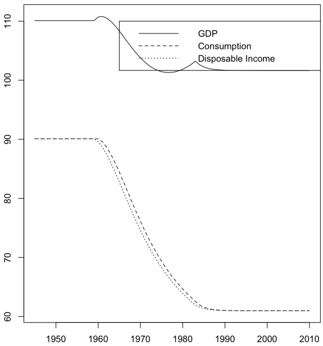
$$\theta = \theta_{-1} + d\theta \qquad d\theta = 4.5\%$$

Implementing the Maastricht treaty III

Assuming (more) rational government

$$G = G_{-1}(1 + gr_g) \qquad gr_g = 3\%$$
$$\theta = \theta_{-1} + d\theta \qquad d\theta = 4.5\%$$





Take home message

- Open source package to simulate SFC models
- Two simple models: SIM and PC
- Role of buffer stocks and expectations
- Difference between short-run and long-run Keynesian multiplier
- Devastating impacts of the Maastricht treaty (Strong assumption of Government expenditures)

References I

W. Godley and M. Lavoie. *Monetary Economics An Integrated Approach to Credit, Money, Income, Production and Wealth*. Palgrave MacMillan, New York, 2007.

Stephen Kinsella and Terence O'Shea. Solution and simulation of large stock flow consistent monetary production models via the gauss seidel algorithm. *Journal of Policy Modeling*, 2010.